Rayleigh-Taylor Instabilities in Thin Films of Tapped Powder

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Abstract

We observe powder "droplets" forming when tapping repeatidly an horizontal flat plate initially covered with a monolayer of fine powder particles. Starting from a simple model involving both the air flow through the porous cake and avalanche properties, we setup an analytical model which satisfactorily fits the experimental results. We observe a close analogy between the governing equations of the phenomenon and the basic physics of wetting liquids, including the equivalent of the Laplace law and the surface tension parameter leading to the well known Rayleigh Taylor instability.

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In the recent years, there has been a great deal of interest in the response of granular materials to various kinds of external perturbations. Up to now, the vast majority of the experimental, theoretical and simulated works have dealt with model granular solids in the sense[1] that the particles were supposed to be large enough (i.e. typically larger than $100\mu m$) to avoid significant interaction with the surrounding fluids[2]. In reverse and rather paradoxically, the understanding of the behavior of fine powders has received much less attention although it is universally recognized as the keystone of an increasing number of high-tech industrial processes. In this spirit, a few recent attempts were made towards the analysis of the behavior of fine (typically in the range from 1 to less than $30\mu m$) cohesive or non-cohesive powders in air (e.g. [3][4][5][6][7]) or of larger particles submitted to excessive windy conditions[2][8], such as in desert dunes[9].

In a recent paper[7] we reported a series of experiments and an analytical model dealing with a thick (about 10mm) slice of fine powder in a container tapped from below. We reported that, under these circumstances, a quasi periodic corrugated pattern spreads out. The characteristic wavelength of this pattern was found to be proportional to the amplitude of the taps. Our theoretical model involved two basic features of the fine granular material : First, the Darcy's law for modelling the air flux trapped under the layer and pushed through the porous cake of granulate when the pile falls down immediately after the shock. Secondly, it involved the maximum stability angle of a granulate before generating avalanches. It was shown that a corrugated surface stands as a more stable state than a horizontal flat surface with respect to air blow from below, because it is easier to eject a particle from a flat surface than from an inclined surface sitting at the avalanche angle. We explained that the apices of the hills created by air blow from below are unstable as compared to both sides of the hills. A couple of further experiments showing direct "crater like" formation resulting from this "volcano effect", provided us a clear view of this last feature.

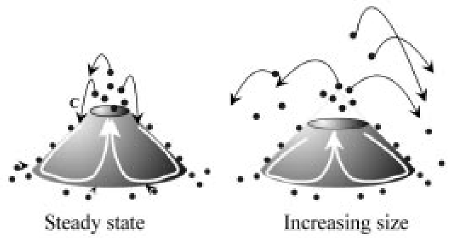

Figure 1: Sketch of the trajectories of the powder particles participating to the intrinsic convection process when the heap are ejected above the plate resulting from either taps or air blowing from below.

Thus, our model explicitly accounts for the earlier Faraday's observation[10] of the "chimney or volcano effect". Faraday stated that " It forms a partial vacuum into which the air round the heap enters with more readiness than the heap itself. The particles of the heap rise up at the center, overflow, fall down upon all slides and disappear at the bottom, apparently proceeding inwards" as sketched in Fig.1 which, additionally shows the resulting inner convections rolls.

Keeping along the same line, we make a step further considering now a thin slice of a fine powder (typical particle size 30 μm) spread out over a flat plate. When gently tapping repeatedly and at constant intensity onto the plate, we observe the formation of a collection of separate rounded conical heaps looking like droplets of powder evenly spread over the plate. The resulting pattern strikingly reminds one of the Rayleigh-Taylor instability illustrated by the droplets structure obtained when turning up a glass plate initially covered with a thin film of a wetting liquid. As we will show in the following, this analogy is not fortuitous. It results from an underlying similarity between the equations governing the wetting properties of liquids and the behavior of powder piles interacting with a surrounding gas.

Several basic characteristic features of the instability of a tapped thin film of powder can be readily observed starting from a simple table-top experiment : Using a small leucite rule equipped with thin spacers, we spread a monolayer slice of powder (silica beads, diameter about $30\mu m$) over a flat glass plate (size 6x9 cm^2). This glass plate is kept horizontal and secured on its periphery using a latex band which allows a certain degree of freedom for up and down motions. Using a small metallic or plastic rod, we knock gently and repeatedly at a very low pace and at a constant intensity over one corner of the glass plate, applying vertically as brief taps as possible. After a few taps (about ten to twenty), the surface, initially flat, smooth and horizontal, separates into a collection of tiny rounded conical heaps looking like droplets similar to those reported in Fig.2. Starting from the same initial conditions but tapping more energetically while keeping the intensity as constant as possible from one tap to the next, induces a pattern with bigger droplets separated by a larger distance. Note that excessive wetness prevents the observation of these surface patterns.

Definitely more reliable information has been obtained in the course of our experiments, using a more sophisticated device. We set a CCD (charge coupled device) camera above the plate in order to record and process the successive patterns ob-

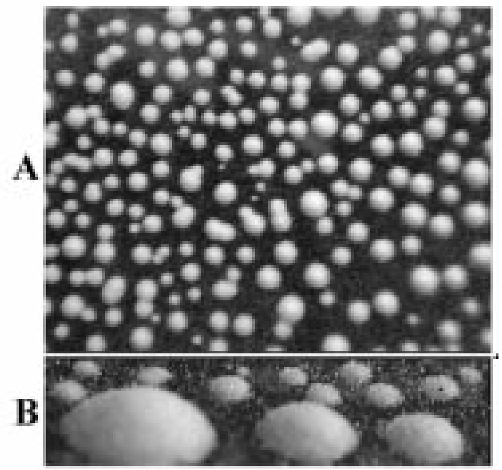

Figure 2: Above : Bird eye view of the pattern obtained after fourty taps over a plate initially covered with an approximately uniform film of powder particles (dia. about 30 microns). The mean distance between neighbouring heaps is about 5mm. Below : The snapshot shows an oblique view of a few small heaps. It exhibits the rounded shape of the apices due to the "volcano effect".

tained during the experiments. Secondly, we used a magnetically driven tapping device and a commercial Bruer and Kjaer accelerometer stuck on the plate in the vicinity of the sample in order to monitor the acceleration of the taps. Thus we get the wavelength dependence on the tap acceleration. Typical experimental results are reported in Fig.3

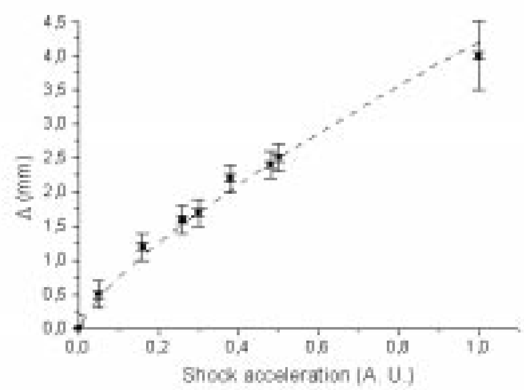

Figure 3: Experimental results obtained with a monolayer slice of silica powder (particle size about 35 μm).The dashed line is a theoretical best fit to Eq. (9)

First we look for a relationship between the height of the approximately identical conical piles and the mean distance separating them. Consider the initial situation when a thin slice of powder of thickness e made of small spherical beads (diameter D) is evenly spread over a horizontal flat surface whose area is S. Suppose now that the powder has been gathered in a number of N disjointed identical conical piles having an angle θ to horizontal and culminating at altitude h. These piles are evenly distributed over the area S. Due to volume conservation, the number N of these piles is approximately given by

$$N = 3\frac{Se\tan^2\theta}{\pi}\frac{1}{h^3} \propto h^{-3} \tag{1}$$

The wavelength Λ of this pattern is the square root of the mean area occupied by each pile

$$\Lambda = \sqrt{\frac{\pi}{3e\tan^2\theta}} \quad h^{\frac{3}{2}} \propto \ h^{\frac{3}{2}} \tag{2}$$

In agrement with Faraday's and more recent authors'[6] observation of the volcano effect and the associated convection, we have shown[7] that when a conical powder pile undergoes a ballistic flight, we can distinguish between two regions, delimitated by a circle at altitude h_C (Fig.1). The lowest region is stable against the upcoming air flux because it is stabilized by the lateral avalanches ($0 \leq h < h_C$). Around the apex we found an unstable part ($h_C \leq h \leq h_T$) (T for top) where grains are expelled by the upcoming air flux. The adimensional parameter C measures the proportion of the unstable part of the heap, so that $C = (h_T - h_C)/h_T$. In our preceding paper dealing with thick powder layers[7], we observed that the stability of the corrugated patterns and the linear dependence of the wavelength on the shocks amplitude are consistent with the fact that C is independent of the shocks acceleration. Stated differently, the steady state of a pattern as sketched in Fig. 1 results from the balance between the number of expelled particles near the apices and the number of particles which are reinjected into the bulk of the heaps at every tap.

In order to eject a single free particle of diameter D and density ρ, sitting on an horizontal surface, we need an air velocity which exceeds the free flight velocity of this particle $v_f = D^2\rho g/18\eta$ where η is the air viscosity and g the gravitational acceleration. Now, we consider a particle sitting at altitude h_C on the inclined surface of the heaps. We built a simplified equation[7] for the screening effect due to the avalanche process, considering that the mass of the particles lying at an altitude h is increased by a factor $Np\sin\theta$, N being the number of the above lying particles pertaining to a single sheet of the inclined granulate and p being the number (typically 5) of sheets possibly involved in the avalanche. Thus the required air velocity needed to eject the considered particle is given by $v_{h_C} = v_f\,(h_T - h_C)\,p\sin\theta/D$.

The velocity of the upcoming air flux at altitude h_C is given by the Darcy's law, so that :

$$v_{h_C} = \frac{K\Delta P}{\eta h_C} = v_f\frac{C}{1-C}\frac{h_C p\sin\theta}{D} \tag{3}$$

where K is the permeability of the powder and ΔP the pressure difference acting over the granular porous cake, due to the air compression when the pile falls down. Thus we find the basic equation governing the problem :

$$\frac{K\Delta P}{Dh_C} = \rho\left(\frac{1}{18}\frac{C}{1-C}p\sin\theta\right)gh_C \tag{4}$$

Written in this form, Eq. (4) can be seen as describing the balance between two antagonistic pressures :

- An "hydrostatic" pressure P_g which accounts for the screening effect of the avalanche properties of the powder

$$P_g = \rho^* g h_C \tag{5}$$

where $\rho^* = \rho \left(\frac{1}{18} \frac{C}{1-C} p \sin\theta \right)$ is the normalized density of the particles sitting near the apices and participating to the avalanches.

- The equivalent of a Laplace-Young pressure, P_l (describing the pressure difference at the interface of two liquids) which can be written

$$P_l = \frac{K\Delta P}{D h_C} = \gamma^* \left(\frac{2}{h_C} \right) \tag{6}$$

where γ^* plays the role of a surface tension and is defined by

$$\gamma^* = \frac{K\Delta P}{2D} \tag{7}$$

In brief, Eq. (4) describes the equilibrium of the analogue of a wetting liquid droplet[11] on an horizontal plate. Thus, we assimilate a conical powder pile with a half spherical wetting material of height h_C and curvature $2/h_C$. This ersatz displays a surface tension (or capillary forces) given by equation 7. This analogue to a surface tension can be seen as resulting from the convective forces (Fig.1) which drag powder particles from the surrounding surface and subsequently inject them into the powder pile. Therefore, the equivalent surface tension of the powder pile has a purely dynamical origin since it results from the convective forces related to the volcano effect. From Eq. 4, we get h_C from the following relationship

$$h_C \simeq \left(\frac{K\Delta P}{D} \frac{1}{\rho^* g} \right)^{\frac{1}{2}} = \left(\frac{2\gamma^*}{\rho^* g} \right)^{\frac{1}{2}} \tag{8}$$

Going on with the analogy to wetting liquids[11], we can also define the usual capillary length λ equating the hydrostatic pressure and the Laplace-Young pressure so that $\lambda = (\gamma^*/\rho^* g)^{\frac{1}{2}} = h_C/\sqrt{2}$ and a related Bond number $Bo = \left(\rho^* g h_C^2 / \gamma^* \right)$

Now, using Eq. (2), we find

$$\Lambda = \sqrt{\frac{\pi}{3e\tan^2\theta}} \left(\frac{K\Delta P}{D} \frac{1}{\rho^* g} \right)^{\frac{3}{4}} = \sqrt{\frac{\pi}{3e\tan^2\theta}} \left(\frac{2\gamma^*}{\rho^* g} \right)^{\frac{3}{4}} \tag{9}$$

Here a numerical estimation of the involved parameters is imperative. We calculate an approximate value for the surface tension γ^*starting from Eq. (9) using typical values for $\Lambda(5mm)$, e$(20\mu m)$ and ρ^* obtained for $C = 5\%$. We get $\gamma^* \simeq 2.3 * 10^{-5} N m^{-1}$ which means that this constant is about 3000 times smaller than the surface tension of pure water. As expected, λ and h_C are in the order of 1mm. Moreover, starting from Eq. (6) we can get an estimated value for the pressure difference between the altitude h_C and the base. First, we consider that the permeability of the granular material is a fraction of the cross sectional area of a single particle. Thus, we get ΔP in the order of 3 Pascal. This quantity should

Wetting liquid	**Eq.**	**Blown powder heap**	**Eq.**
Surface tension	$\gamma = \frac{dF}{dl}$	Convective forces	$\gamma^* = \frac{K\Delta P}{2D}$
Droplet radius	R	heap height	h_C
Laplace-Young law	$\Delta P = \frac{2\gamma}{R}$	Eq. 6	$\Delta P^* = \frac{2\gamma^*}{h_C}$
droplet equilibrium	$\frac{2\gamma}{R} = \rho g R$	blown heap equilibrium	$\frac{2\gamma^*}{h_C} = \rho^* g h_C$

Table 1: Basic equations for a wetting liquid and a blown powder

be a fraction of the maximum possible air pressure due to the total weight of the powder pile leaning on the basis surface S. This maximum air pressure is found to be about 10 Pascal which therefore stands as a correct order of magnitude.

Table 1 summarizes the analogy between the basic equations governing the powder heap equilibrium and the equations governing the equilibrium of liquid droplets.

Now, starting from this analogy and using e.g. Eq. (4), we can transcribe the classical demonstration of the Rayleigh-Taylor instability for wetting liquids. The standard analysis consists in examining the evolution of an infinitesimal sinusoidal distortion of the initially flat surface. Note that the basic calculation for liquids (found in text-books) leads to a wavelength dependence $\Lambda \propto (\gamma/\rho g)^{\frac{1}{2}}$. Here, the distortion is by no means infinitesimal. We rather introduced the volume conservation condition (Eq. (1)) which leads to $\Lambda \propto (\gamma^*/\rho^* g)^{\frac{3}{4}}$. But except for this difference, the underlying phenomenology of the blown powder mimics the standard Rayleigh-Taylor instability.

Still proceeding with the analogy of the inner pressure within a powder heap given by Eq. (6) which mimics the Laplace-Young law, we predict that if two powder heaps of unequal sizes are sitting next to each other, the smaller one would be sucked into the larger one just as this occurs between two communicating bubbles. The experimental result fulfills this assertion as can be seen in Fig.4.

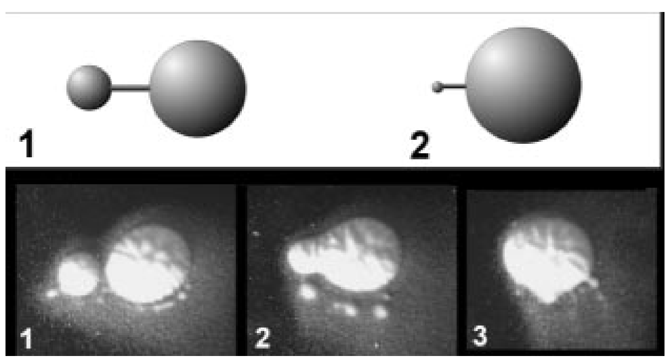

Figure 4: Above, sketch of a basic experiment showing that the inner gas pressure is larger in a smaller droplet. When two bubbles are connected by a small pipe, the small bubble is sucked into the largest one. Below, the bird eye view of an experiment showing the fusion mechanism among tapped powder heaps. The smaller heaps are sucked into the largest heap in agrement with eq. 4

Furthermore, it is well known that a thin film of a wetting liquid spread over a cylindrical rod is intrinsically unstable. Due the so-called Rayleigh instability, such a cylindrical film splits into a collection of approximately equally spaced droplets because of the curvature of the supporting surface. Here, we find a seemingly identical behavior starting from a single line of powder spread over a horizontal plate (Fig.5).

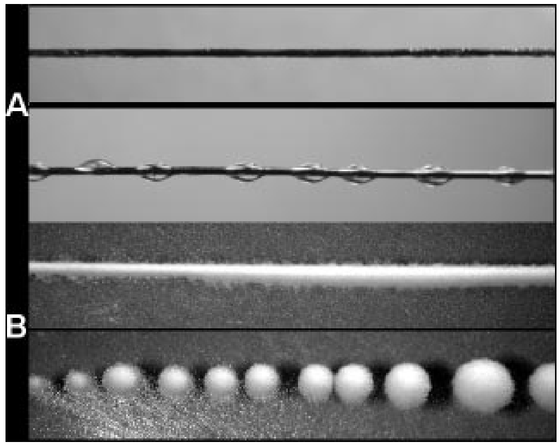

Figure 5: Fig. A is a snapshot of the spontaneous droplet formation when a thin rod is covered with a wetting film of maple syprup (above). The liquid films splits into small droplets (below) Fig. B is a bird eye view of an seemingly analogous behaviour using a fine powder. Starting from a line of fine powder (above), the support is tapped, the thin line splits into quasi regularly spaced powder piles (below).

However, the analogy is misleading here because the curvature of the support does not play any role in this experiment. We should rather consider this result as a 1D version of the Rayleigh-Taylor instability in fine powders.

At this point, it is meaningful that the ambient fluid viscosity η cancels out in Eq. (4) just as in the classical analysis of the steady state of the Rayleigh Taylor instability. It is also certainly not fortuitous that the similarity between the leading equation for wetting fluids and blown fine powders comes from the formal analogy between the Darcy's and Laplace-Young law. In connection with that, we note that the equivalent surface tension γ^* is in the order of $D\Delta P$. The corresponding energy due to a small surface change dS reads $\delta W \sim \gamma^* dS \sim D^3 \Delta P \sim V_p \Delta P$ where V_p is the volume of a single pore of the granular cake. This merely illustrates the fact that the Darcy's law expresses the Poiseuille law across a single pore of the porous cake.

Even if it has the merit to establish a connection between the (yet unknown) description of blown powder properties and the (already known) wetting liquids behavior, our simple theoretical explanation certainly lays itself open to several criticisms. In particular, it does not convey any information regarding the development of the surface instability. Such an analysis would involve the introduction of a sort of powder viscosity[12] which is not considered in the present model dealing with the steady state of the process. A time resolved scrutiny of the pattern growth would probably convey information about this question. We postpone the description of this study to a forthcoming paper.

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